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NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

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EMPIRICAL CORRECTIONS TO THE SPAN LOAD DISTRIBUTION AT THE TIP

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SUMMARY

An analysis of existing pressure-distribution data was made to determine the variation of the tip loading with wing plan form. A series of empirical tip corrections was derived that may be added to theoretical curves in certain cases to obtain a closer approach to the actual loading at the tip.

The analysis indicated that the need for a tip correction decreases as either the aspect ratio or the wing taper is increased. In general, it may be said that, for wings of conventional aspect ratio, corrections to the theoretical span load curves are necessary only if the wing is tapered less than 2:1 and has a blunt tip. If the tip is well rounded in plan form, no correction appears necessary even for a wing with no taper.

INTRODUCTION

The recent trend toward the use of airfoil theory for determining the load distribution for structural design arose principally because it was found that the various combinations of wing taper and wing twist that were being used called for a rational system of specifying the load distribution. Although it was known that the lifting-line theory gave load distributions that were, in general, in good agreement with those experimentally obtained, it did not indicate the presence of a tip effect that was known to exist for certain wing shapes.

The results of flight tests, reported in reference 1, indicated that, contrary to theory, the distribution of the normal-force coefficient for a rectangular wing was practically independent of the tip plan form when no twist

was built into the wing and also that the shape of the distribution curve varied with the wing lift. This result is also verified by wind-tunnel tests of untwisted rectangular wings as shown by figure 1. Figure 1(a) shows typical wind-tunnel distribution curves for a rectangular wing with a square tip; figure 1(b) shows corresponding curves for a wing with a circular tip. The dotted curves show that the theoretical values of c_n at the tip increase rather rapidly as the rounding proceeds from the blunt to the more slender circular plan form. Also it can be seen that the shape of the theoretical curves does not vary with wing C_N as the ratio of the ordinates of any two curves is equal to the ratio of the wing C_N values.

As a result of the foregoing discrepancies, some doubt still accompanies the use of the theory when applied to structural design. The present note is therefore intended to supply information concerning conditions under which a tip correction may be required and, when one is required, to estimate its character and magnitude. The corrections and correction factors given herein do not pretend to great accuracy since, in some instances, the experimental data available did not permit the establishment of accurate quantitative values. Since the errors involved in the present methods of structural analysis of airplane wings may easily be of the same order as the corrections, however, the fact that some of the factors may not be of great accuracy is of minor importance.

DERIVATION OF TIP CORRECTIONS

Load Distribution

Since the values of c_n at the tip are, for a given wing C_N, lowest when there is no rounding at the tip, it was decided to base the following empirical corrections on theoretical curves for wings with straight tips. The first step was to determine the differences between the actual experimental curves for rectangular wings, regardless of tip shape, and the computed theoretical curves for similar wings with a rectangular tip. These differences were plotted against span location for various values of wing C_N and for each wing aspect ratio. By averaging these difference curves, for a given aspect ratio and at a given wing C_N, zero lift c_n distribution curves were obtained.

These curves were all characterized by small negative increments over the larger portion of the span except near the tip region where the increments became large and positive. When the small negative increments along the span were neglected, it was found that the distance in from the tip at which a pronounced tip effect appeared was roughly 40 percent of the mean chord (0.4 S/b) regardless of aspect ratio. This fact is illustrated by figure 2, which shows distribution curves taken from reference 4 for aspect ratios 3 and 5. It can be seen that the distance in which a tip effect appears tends to be constant and that the shape of the increment curves would be substantially the same.

The empirical corrections found by the foregoing procedure are shown in figure 3(a) for the rectangular wing of aspect ratio 6. These increments are to be added to the theoretical curves for rectangular wings with square tips, starting at a distance of 0.4 S/b in from the tip.

A comparison of the increment curves for aspect ratio 6 with those for other aspect ratios showed that, instead of having separate sets of curves, a factor might be used to convert the corrections of figure 3(a) to other aspect ratios. This factor, which is shown in figure 3(c), serves as a multiplier to the ordinates of figure 3(a). The variation of this factor was determined mainly from the data contained in references 1 and 4.

The procedure used to determine the increments for tapered wings was the same as that used for rectangular wings. The experimental data on tapered wings were, however, much more limited in scope, being confined for all practical purposes to those given in references 5 and 6, inasmuch as data from other sources generally contained unknown amounts of twist near the tip portion of the wing. This unknown twist was caused by rounding the tip portions of wings that were originally trapezoidal in form. Comparisons for the series of wings tested in reference 5 indicated an agreement between theory and experiment, within the experimental error, for the 2:1 and 5:1 tapered wings and, hence, it may be inferred that no correction would be necessary for taper ratios greater than 2:1. Similar comparisons of the tapered wings of reference 6, however, indicated that there should be a small correction for the 2:1 taper. The final correction-factor curve for taper (fig. 3(c)) shows the variation decided upon as the best average. Since the one test available for a taper greater

than 2:1, i.e., the 5:1 taper of reference 5, showed little disagreement between experimental and theoretical distributions below the stall, the correction factor for this taper was assumed to be zero.

In order for the final tip corrections to fit the experimental trends, i.e., for the tip corrections to decrease with increase in aspect ratio and to disappear as the taper increases, the correction factors for aspect ratio and taper must be multiplied together before being used to expand or contract the ordinates of the curves given in figure 3(a).

Although the effect of sweepback may theoretically be considered as equivalent to a washin and sweepforward as a washout (reference 7), this trend could not be definitely determined from the experimental data available. Pressure-distribution results from reference 8 for wings with both 10° and 20° sweepback and sweepforward seem to substantiate the theoretical trend in the load variation for the portions inboard of the tip. In the tip region (0.4 S/b), however, the experimental results are apparently opposite to the trend indicated by the theory. This behavior may be due to the test procedure employed as the wing was simply rotated about a yaw axis in the reflection plane to give the desired amount of sweep. This procedure caused the pressure ribs to be at an angle of yaw with respect to the air stream and also caused the tips to become raked in an uncommon manner. Other tests (references 5 and 6), in which the wing axis (the line joining the quarter-chord points) was bent, indicated that the effect of ordinary amounts of sweep on the span loading may be neglected. For tailless airplanes with large amounts of sweepback, the corrections would not apply.

Moment Distribution

The experimental tip effect, however, is not entirely confined to an increase in the tip loading but is accompanied by a considerable increase in the section moment coefficients. In the present report the increase in section moment was empirically determined by analyzing various sets of data from references 1 to 6 and 8 to 10 for the value of Δc_m occurring in the expression

$$c.p. = a.c. - \left(\frac{c_{m_0} + \Delta c_m}{c_n}\right)$$

where c.p. is section center of pressure

a.c., average aerodynamic center for sections inboard of the tip distance

c_{m_0}, section moment coefficient at zero lift

c_n, section normal-force coefficient

The values of Δc_m obtained from the foregoing equation were plotted against percentage of tip distance (0.4 S/b) with section c_n as a parameter. The final averaged curves are given in figure 3(b) for a rectangular wing of aspect ratio 6. The correction factors for taper and aspect ratio (fig. 3(c)) previously found for the tip-load increments also apply for the moments.

APPLICATION OF EMPIRICAL CORRECTIONS

The basic curves to which the Δc_n corrections are added are the theoretical c_l or c_n* curves for the particular aspect ratio and plan form used; the tips, however, are considered to be straight. The theoretical curves may be determined by any of the various methods available, for example, by those methods in which the lift is expressed as a Fourier series. If the actual wing has linear taper other than the rounding at the tip, the uncorrected lift distribution at a C_L of 1.0 may be obtained from figures 4(a), 5(a), and 6(a), which give theoretical curves for tapered wings with straight tips. Figures 4(b), 5(b), and 6(b) give the zero lift distributions for wings with a linear twist.

The addition of the tip increments to the curve of c_l distribution changes the wing C_L, or C_N, to a slightly different value from that originally used and, consequently, a small correction must be introduced. It is necessary that the final distribution for a definite value of the wing C_L satisfy the equation

*In this note, wing C_L and wing C_N are considered to be equivalent as are section c_l and section c_n.

$$C_L = \frac{1}{S} \int_{-b/2}^{b/2} c_l \, c dy$$

Since the correction to the total wing C_L is generally very small, it may be made as follows:

(1) Convert the resultant span c_l curve, after the corrections are added, to a load curve and integrate graphically to find the total load.

(2) Determine the ratio of the desired load (corresponding to the original or desired value of C_L) to the load of step (1).

(3) Multiply the ordinates of the load curve of step (1) by the ratio of step (2) and, if desired, convert back to a span c_l curve.

For the linearly tapered wing with either a straight or a circular tip, the following shorter method may be used. The basic theoretical span loading is obtained for an initial value of $[C_L]$, which is different by the amount contributed by the tip increment, instead of for the final desired value of C_L. This value of $[C_L]$ is given by

$$[C_L] = C_L - F_1 \; F_2$$

where F_1 and F_2 are factors that may be obtained from figure 7. The addition of the empirical tip correction will then bring the resultant C_L up to the desired value, within the limits of precision obtained in a graphical integration. The curves of figure 7 may be interpolated for other tip shapes if desired.

COMPARISONS WITH EXPERIMENT

Several comparisons between theoretical, experimental, and theoretical curves with the tip increments added are given in figure 8. These figures are largely self-explanatory and indicate that the modified curves show a reasonable agreement with the experimental curves over a wide range of conditions.

From the comparisons indicated in figures 8(c) and 8(d) it may be concluded that, ordinarily, it is immaterial whether or not the tip correction is included with the theoretical curves if the taper ratio is greater than about 2:1. Comparisons of the shape of the experimental and theoretical c_l curves for a straight wing with a circular tip (figs. 1 and 8(b)) also indicate that no positive gain in accuracy would be had by using tip corrections when it is known that, in practice, the actual c_l distribution will be influenced by extraneous factors about which there is little information. Similarly, for tapers between 1:1 and 2:1, no correction would presumably be required to the theoretical curves if the tips were well rounded. Thus, the only time that a tip-load correction would be necessary on a wing with little or no sweepback, if theoretical distribution were used in design, is when the wing has a small taper in a combination with a blunt tip.

Even though it may be unnecessary to apply a correction to the load, it may be necessary to consider the effect of an increase in section moments at the tip, particularly if the design condition is at a fairly high wing C_L, because in certain types of construction more load would be thrown on the rear spar at the tip.

The comparisons made between the computed and experimental curves, which include data other than those shown in figure 8, are summarized in figure 9. The curves for this figure were obtained by plotting the differences between corrected theoretical and experimental curves at all points along the span for various aspect ratios and wing taper ratios. Positive differences indicate that the computed values are high and vice versa. It will be noted that the differences as a whole are approximately symmetrically disposed about the zero axis and are indicative of the averaging that was necessary in deriving the increments. It can be seen that for monoplanes without twist the agreement is good throughout the span. Although the discrepancies are larger for the biplane and the monoplane with 15° twist, it must be remembered that they represent fairly extreme cases.

When it is necessary to use a correction to the loading, the specific steps to be employed in applying the derived corrections could be as follows:

(1) From the conditions of the problem determine the wing C_N or C_L based on a wing area assumed to carry through the fuselage.

(2) Determine both the aspect ratio and taper ratio of the wing, considering the tip as straight.

(3) From results given by steps (1) and (2) determine: factors F_1 and F_2 and find $[C_L] = C_L - F_1 F_2$

(4) If the taper is linear, the theoretical c_l distribution may be found from figures 4(a) to 6(a). Multiply section c_l values by the value of $[C_L]$ from step (3) and add twist curves (figs. 4(b) to 6(b)) reduced or increased in proportion to the actual twist.

(5) Determine the tip corrections from figures 3(a) and 3(b). The distance affected by the tip is 40 percent of the mean chord. The tip increments are modified by the aspect-ratio and taper-ratio factors (fig. 3(c)).

(6) Add the tip increments to the curves of step (4) and reduce to a load curve.

For special cases where double taper or an odd twist occurs, the tip corrections may presumably be applied as before to the theoretical curves, but the factors F_1 and F_2 are no longer applicable and an integration will be required after adding the corrections to determine the new C_L value. The tip-moment increments (fig. 3(b)) are to be added to the basic section-moment coefficients; first, however, a correction should be made to the increments for the effect of aspect ratio and taper.

For wings with well-rounded tips, the appropriate load distributions may be obtained directly from references 11 and 12, as no tip corrections to the load are required.

Langley Memorial Aeronautical Laboratory,
National Advisory Committee for Aeronautics,
Langley Field, Va., July 13, 1937.

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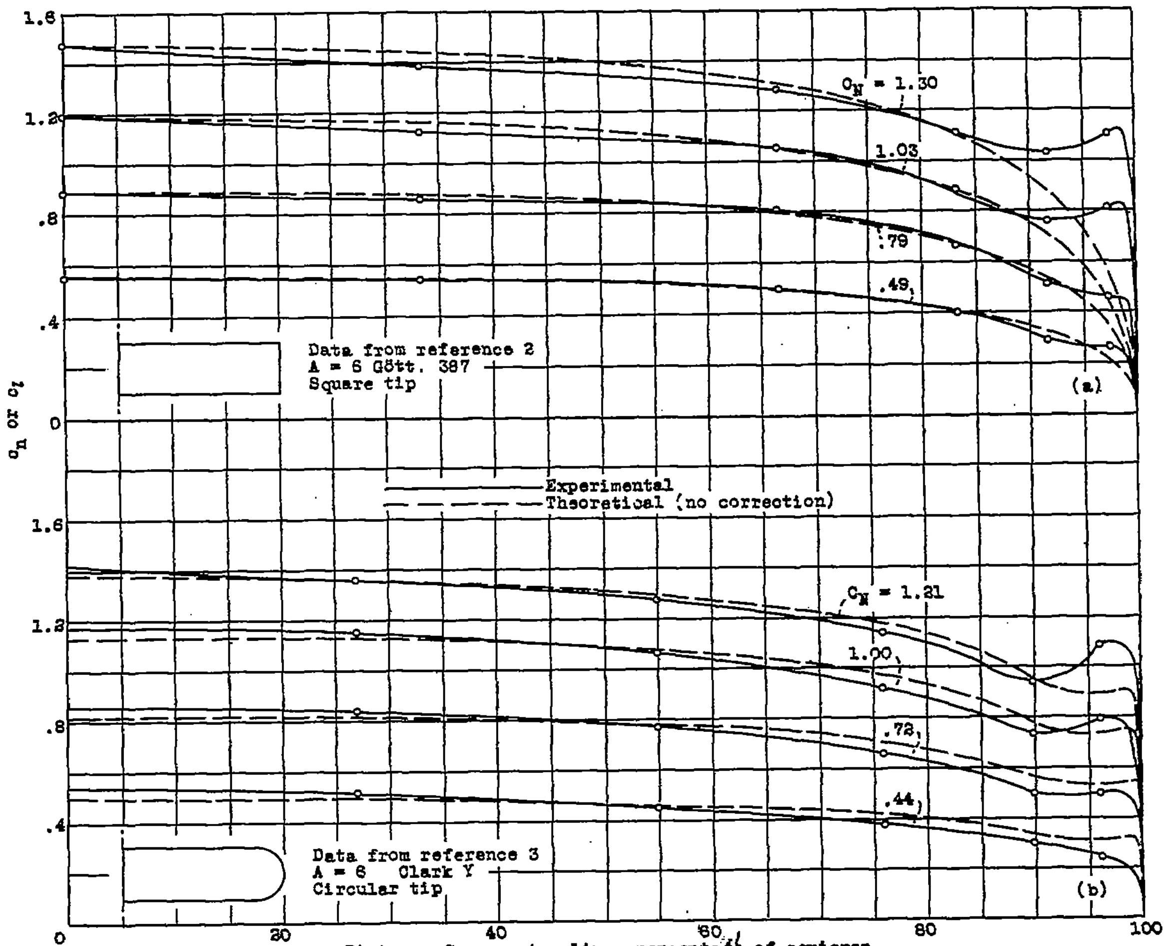

Figure 1. - Experimental and theoretical distributions for rectangular wings.

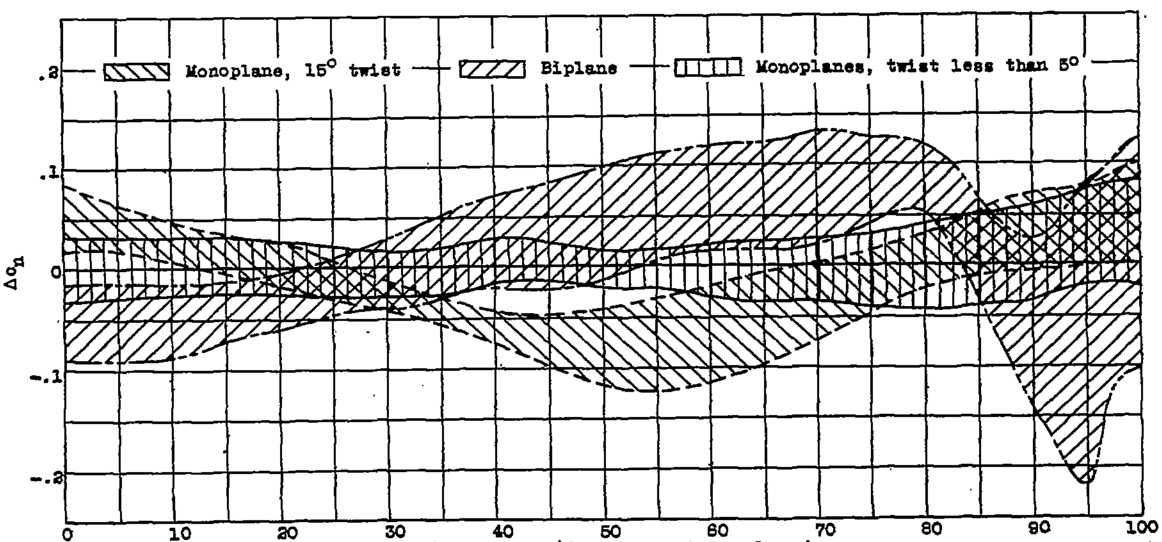

Figure 9. - Summary of comparisons between computed and experimental span load distributions

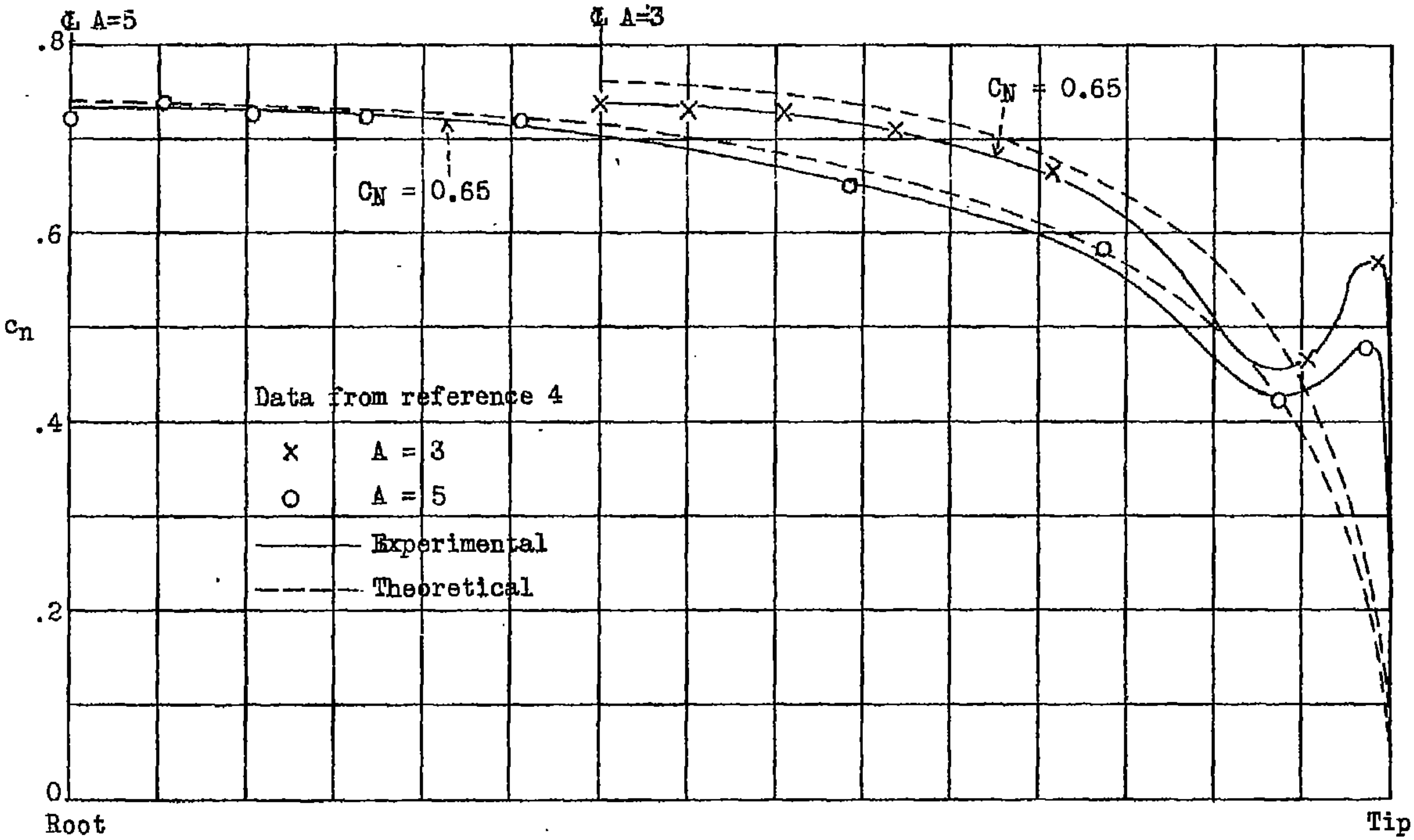

Figure 2.- Influence of aspect ratio on tip c_n distribution.

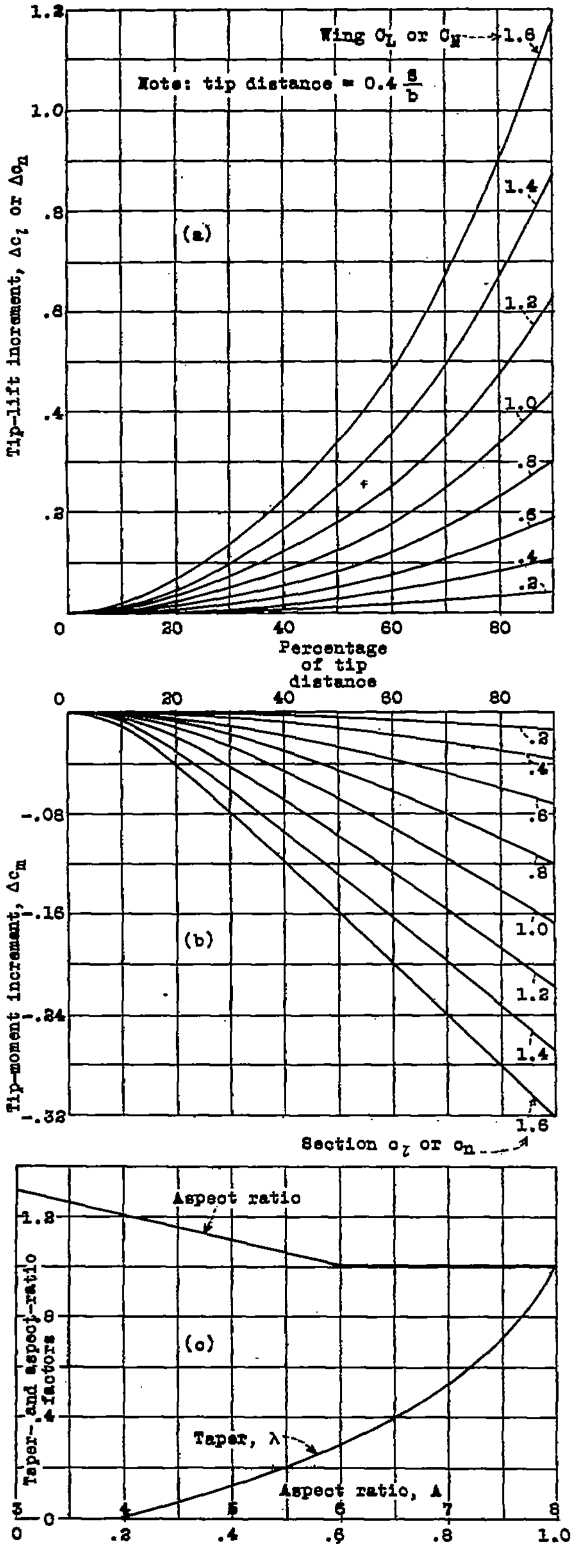

Figure 3.- Tip corrections.

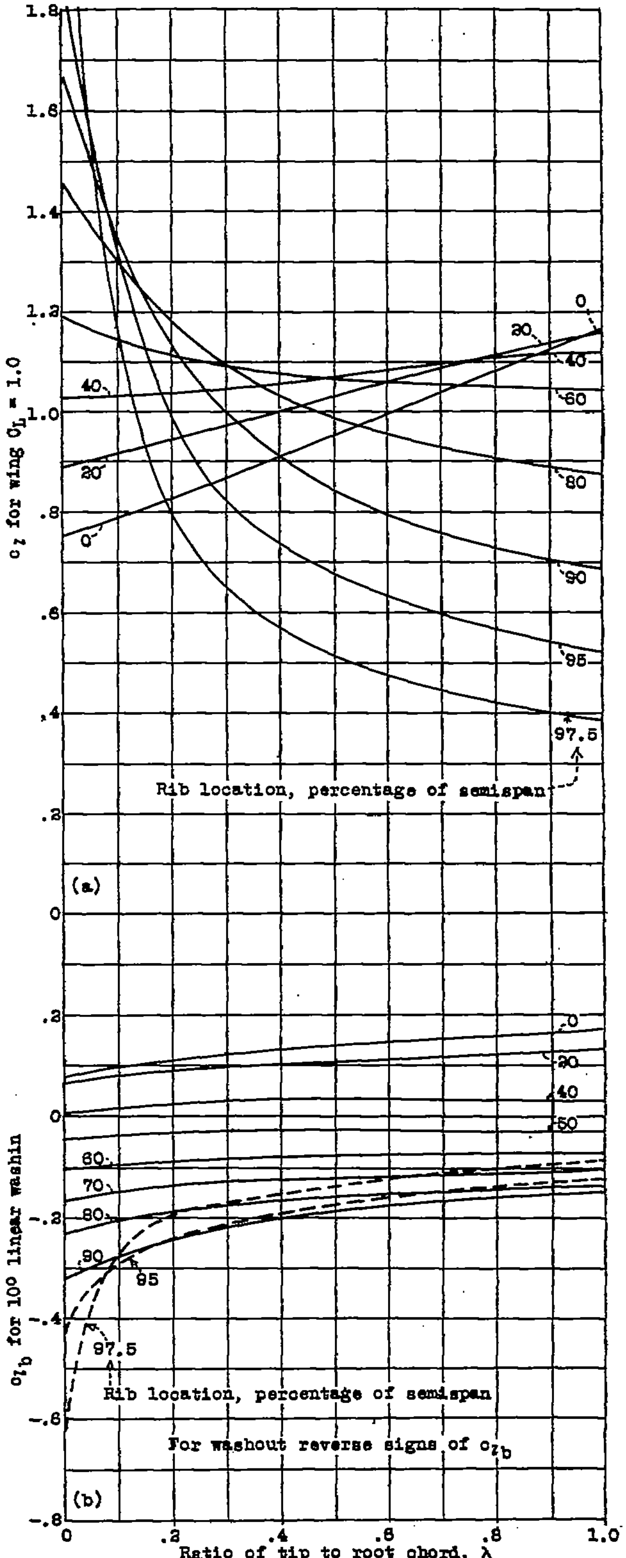

Figure 4.- Theoretical span c_l and c_{l_b} distributions for wings with straight tips. A = 4.

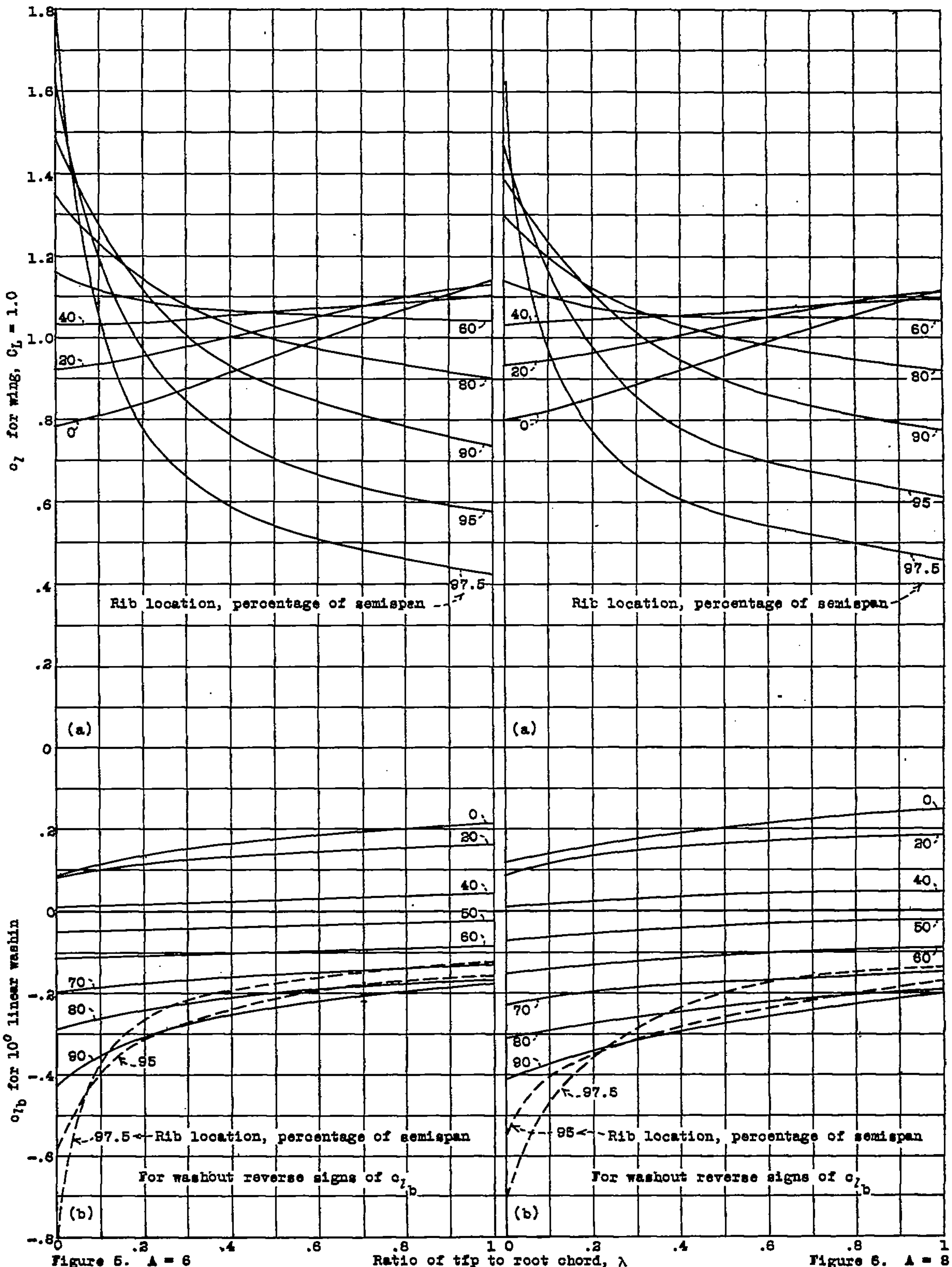

Figure 5. A = 6 Figure 6. A = 8

Theoretical span c_l and c_{l_b} distributions for wings with straight tips

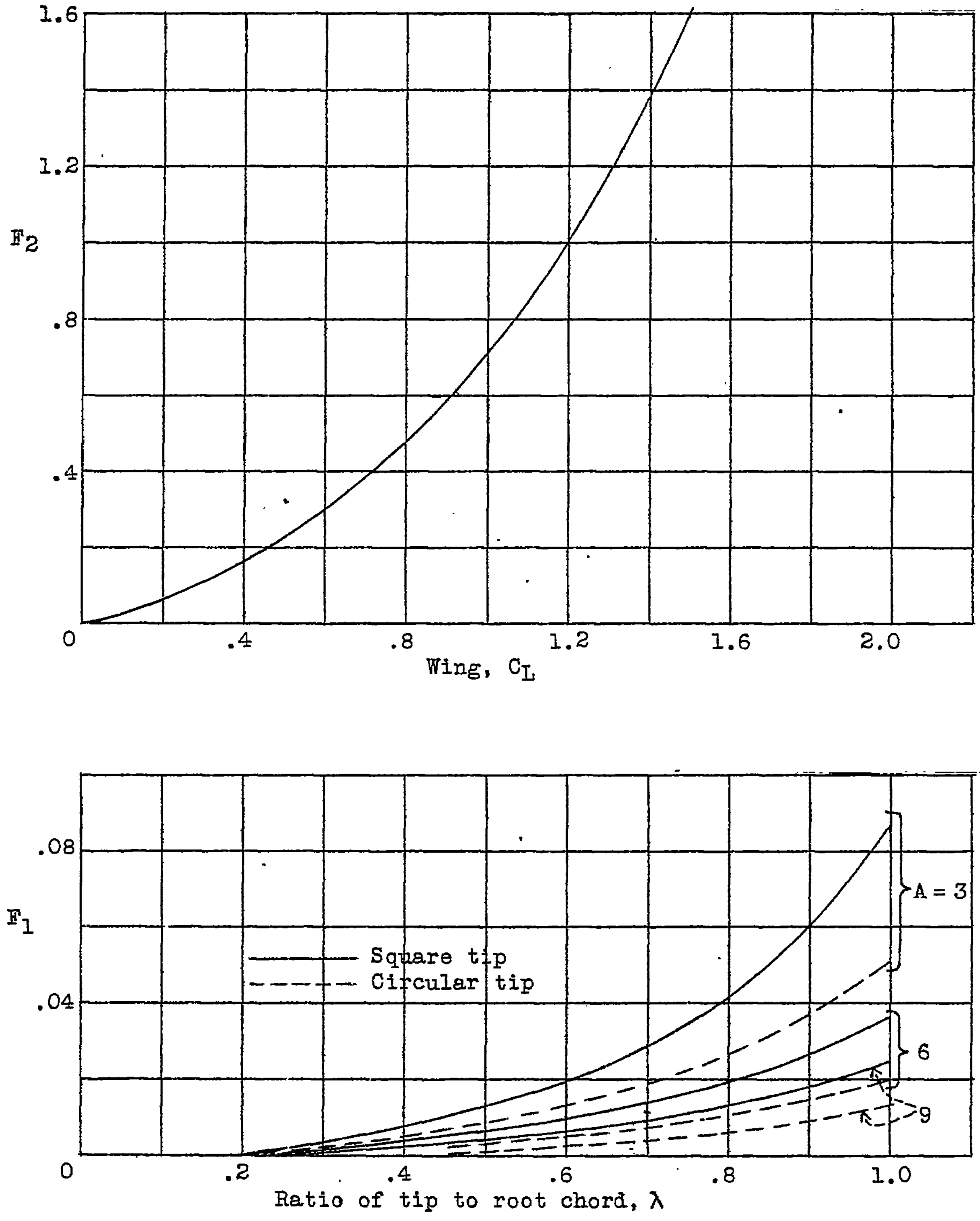

Figure 7.- Correction factors for wing C_L.

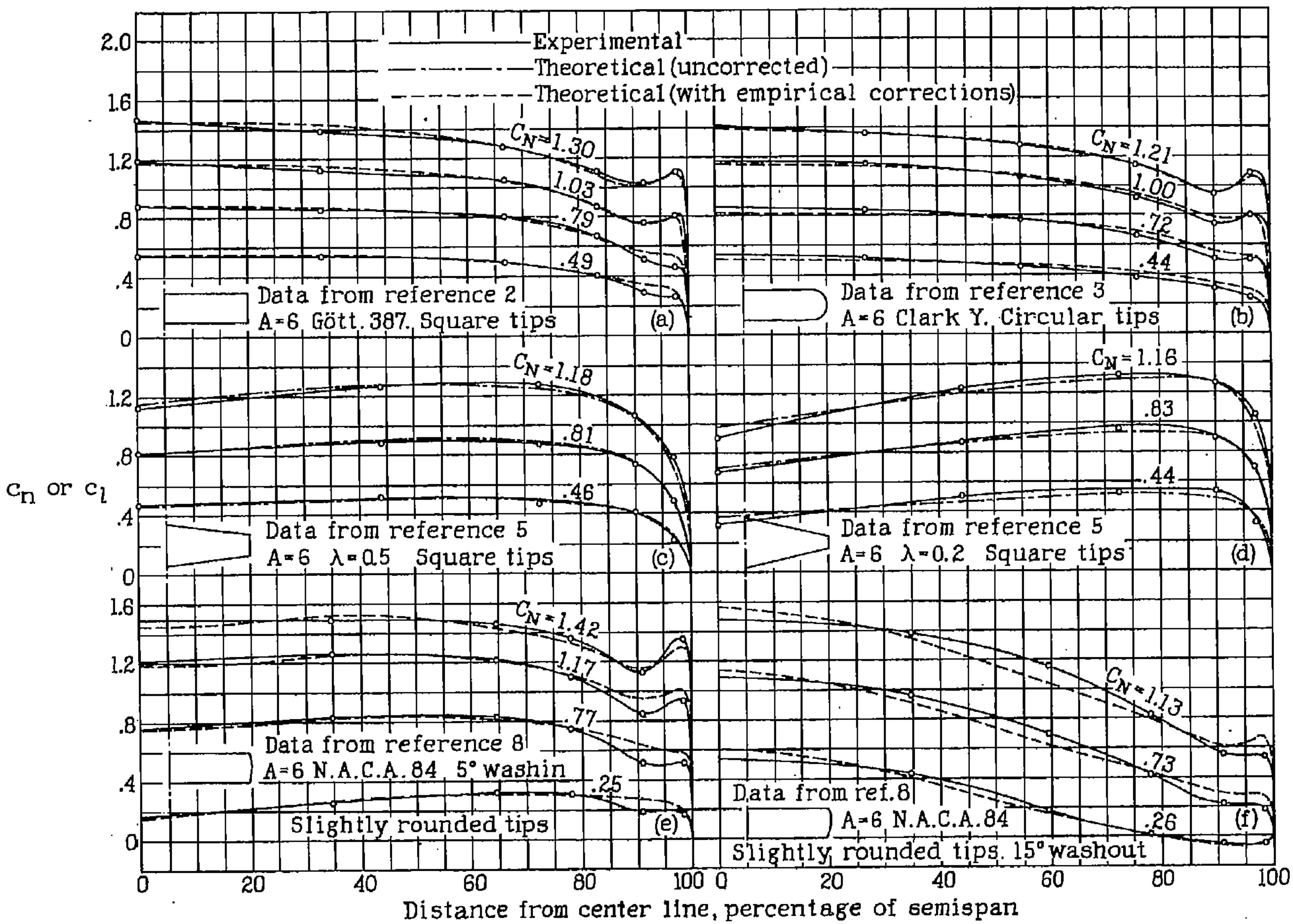

Figure 8.-Comparisons between computed and experimental curves